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Statistical Interpretation of the Gradation Acceptance Criteria of Granular 'A' Aggregates

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Glossary Of Terms

Arithmetic mean - ($\bar{X}$; $\bar{\bar{X}}$; $\bar{X}'$) - The measure of the central tendency. $\bar{X}$ = the mean of a set of observations (e.g. a lot) defined as the sum of all the observations divided by the number of observations (n). $\bar{\bar{X}}$ = the average of a set of $\bar{X}$ values.

$\bar{X}'$ = the mean of the population, which is defined as the true value of the central tendency. The more the number of observations the closer is $\bar{X}$ to $\bar{X}'$.

Assignable cause - An error usually due to process change, causing a significant variation (e.g. a hole in a testing sieve, a change in the pit face etc.).

Average range - ($\bar{R}$) - The sum of the ranges of the lots divided by the number of ranges.

Buyer's risk (β) - The probability of accepting poor or unacceptable material. Commonly referred to as Type II or β error.

Chance cause - An event occurring by accident or at random.

Control charts - A graphical method of process control which detects when assignable causes are acting on a process.

Degrees of freedom (ν *[nu]*) - The number of free choices. e.g. if we know the mean of the lot (4 tests) we can freely choose three sublot values, the fourth sublot value is fixed by the known lot mean. Consequently the degree of freedom of four observations (n) is three; $\nu = n-1$.

Dispersion (i.e. spread) - The extent to which the data are scattered about the central tendency.

Lower control limit (L.C.L.) - A process control criterion associated with the control charting technique. (e.g. in gradation control charts for the mean of the lot the L.C.L. is the same as the lower limit of the specification band). U.C.L. = upper control limit.

Normal distribution curve - A curve being a bell shape, that is determined by values of $\bar{X}'$ and σ' and is often used to describe the distribution of individual measurements.

Population - Any set of individual measurements having some common observable characteristic. (e.g. all the individual gradation test results of a base course aggregate, produced by one plant and used on one contract).

Quality assurance - A management method intended to evaluate the quality of materials and/or construction through an unbiased inspection system which studies the quality control and acceptance sampling and testing procedures being used.

Quality index (Q_u & Q_L) - An empirical expression, the values of which are utilized for estimating the percentage of a lot within specification limits.

Range (R) - The difference between the largest and smallest measurements in a set of data.

Seller's risk (α) - The probability of having acceptable material or construction rejected. Commonly referred to as Type I or α error.

Standard deviation (σ) - A term measuring dispersion. Calculated from the square root of the differences between the individual measurements in a group and their average. σ = the standard deviation of a set of values (say a lot or lots); σ' = the standard deviation of the population.

Variance (σ^2) - A measure of dispersion found by squaring the standard deviation.

Statistical Interpretation of the Gradation Acceptance Criteria of Granular A Aggregates

(1) Introduction

The purpose of this treatise is to explain the gradation acceptance requirements of MTC granular A aggregates by means of rudimentary statistics. The gradation acceptance guidelines (1979 & 1980) and the new special provision No. 331 were developed by using some statistical probabilities. For the sake of clarity, however, reference to any statistical interpretation was purposely avoided when these requirements were drawn.

With the establishment of the Quality Assurance Systems Committee and its objective to develop quality assurance specifications, it is felt that a better understanding of the statistics of these specifications is essential.

An attempt is made to explain how the various acceptance requirements were set, what these requirements mean statistically and what types of controls are available to assess the various test results.

The rationale followed throughout the text is easily understandable. It is, however, assumed that the reader has had some previous exposure to basic statistics such as Demonstration Project 42 or the course on Statistical Quality Control presented by the Sigma Partnership in April, 1982.

Special Provision No. 331 stipulates that:

> *A lot of Granular A will be deemed to meet the specification requirements for gradation if the mean of the four test results of the lot is*

within the gradation limits, and the range of the test results of the lot is equal to or less than the maximum acceptable range.

Gradation and range limits are given in the special provision. Here we have two basic requirements for acceptance, namely the lot mean ($\bar{X}$) and the lot range (R_{max}). Let us then see what these requirements mean statistically.

(2) Statistical explanation of the mean and range criteria (4.75 mm sieve)

Acceptable $\bar{X}$ and R_{max} of the lot is given for all the designated sieves in the special provision. The principles will be adequately explained by going through the exercise using one sieve only. For this exercise let us use the 4.75 mm sieve; the same rationale applies to the rest of the sieves.

On the 4.75 mm sieve the special provision specifies that all the lot means should fall between 35% and 55% passing in order to be acceptable. This requirement may be re-phrased in statistical terms thus: the distribution of the sample means ($\bar{X}$) of the size n = 4 shall be between 35 and 55%. And by assuming normal distribution we can further state that all the lot $\bar{X}$ values shall be between the upper limit of $\bar{X}' + 3\sigma_{\bar{X}}$ and the lower limit of $\bar{X}' - 3\sigma_{\bar{X}}$. The symbol $\sigma_{\bar{X}}$ is the standard deviation of the sample means of the size n>1, called the standard error of the mean and expressed by the formula:

$$\sigma_{\bar{X}} = \frac{\sigma'}{\sqrt{n}} \qquad (1)$$

where σ'= the standard deviation of the population and "n" is the sample size. To illustrate the above see Figure 1.

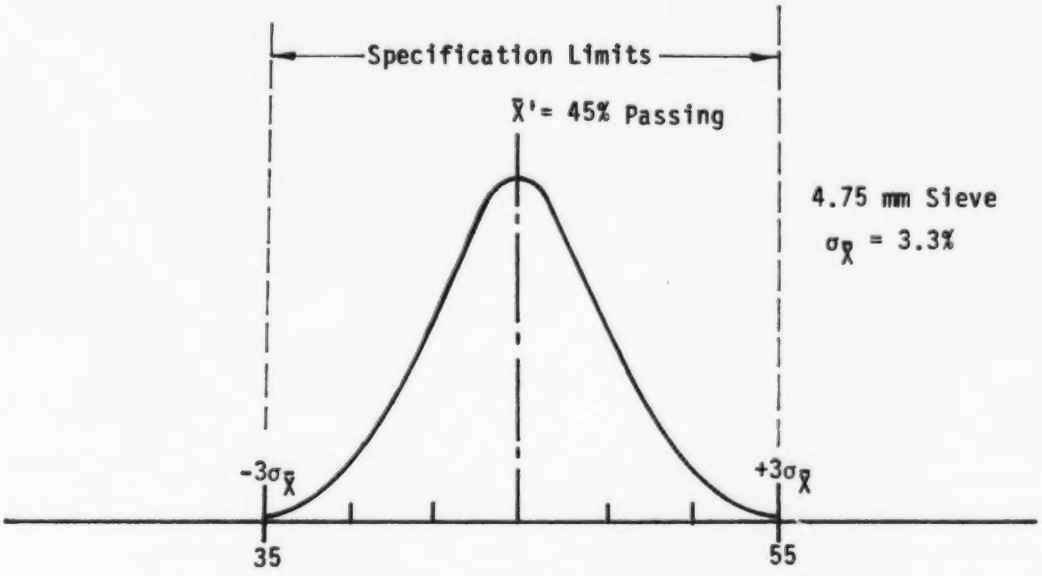


Figure 1
Distribution of Sample Means

Since $\pm 3\sigma_{\bar{X}}$ is equal to $\pm 10\%$ then $\sigma_{\bar{X}} = \frac{10}{3} = 3.3\%$.

According to the lot mean acceptance criterion therefore it is assumed that if the population $\bar{X}'$ is exactly at the middle of the specification band all the lot means will vary between $\bar{X}' \pm 3\sigma_{\bar{X}}$. If the $\sigma_{\bar{X}}$ is equal to or smaller than 3.3% then all the lots will be acceptable (1 in 100 may not be).

From the $\sigma_{\bar{X}}$ we can compute the population σ' and examine the distribution of the individual tests. From equation (1)

$$\sigma' = \sigma_{\bar{X}}\sqrt{n} = 3.3\sqrt{4} = 3.3 \times 2 = 6.6\%$$

Assuming again normal distribution we can say that approximately 68% of the individual tests will lie between the limits of $\bar{X}' \pm \sigma' = 45 \pm 6.6\%$. Some 95% will be between $X' \pm 2\sigma'$; and 99.7% between $\bar{X}' \pm 3\sigma'$. The following figure depicts the dispersion of individual tests (Figure 2).

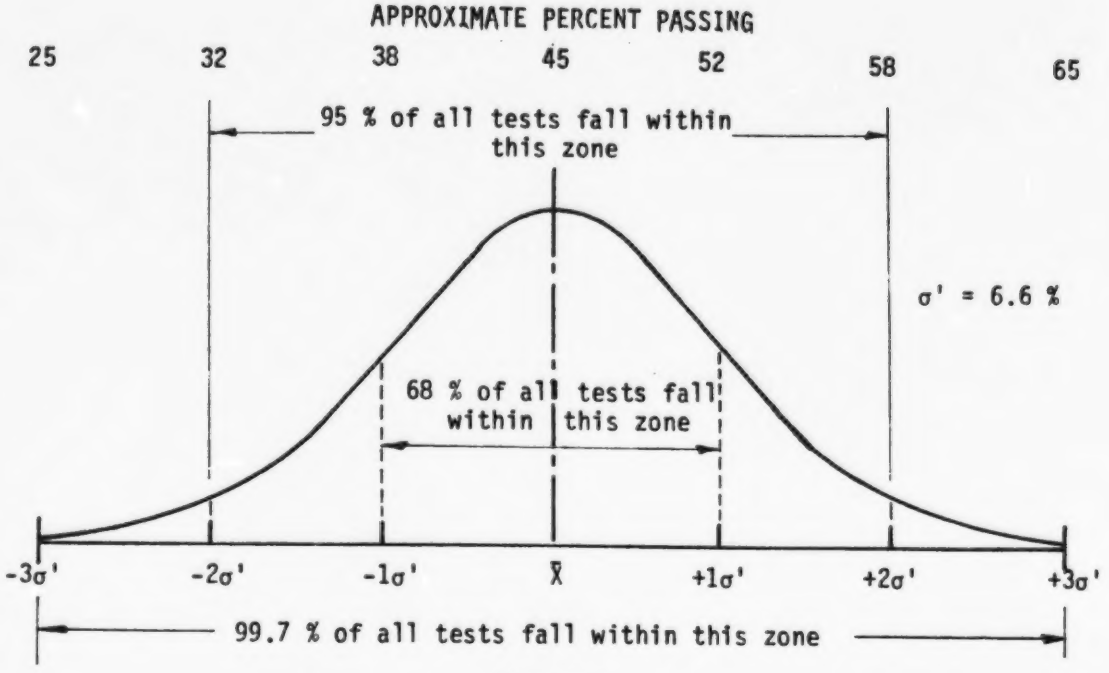


Figure 2
Distribution of Population

Individual test results of fully acceptable lots may fall then between the limits of 25% and 65%. Put otherwise it can be stated that there is a 5% probability that individual test results will fall between 25 and 32% or 58 and 65%, even if all the lot means were within the specified 35-55% limits.

We have to realize that our fixed gradation band specification assumes that the target value (the population mean) must be exactly at the middle of the band and the standard deviation must not exceed the value explained above, in order that all the lots be accepted with full price.

If the population mean is not exactly at the middle of the gradation band, but within the upper and lower specification limits, than the dispersion of the test results (i.e. σ') must be less than computed earlier, in order to conform to the requirements.

The mean of the population and the sample mean (lot means) are the same. To determine the population mean with accepted accuracy we need at least 100 individual tests or 25 lot values. If some historical data is available to determine the sample mean,

then the acceptable dispersion can be easily calculated. Let us assume that a plant produces granular A having $\bar{X}' = 50\%$. The allowable dispersion will be:

$$\bar{X}' = 50\% \therefore \text{upper specification limit} = 55\%.$$

$$55 - 50 = 5\% = 3\,\sigma_{\bar{X}};\ \frac{5}{3} \simeq 1.7 = \sigma_{\bar{X}}$$

If the standard error of the mean is 1.7% then the total dispersion will be: $\bar{X}' \pm 3\,\sigma_{\bar{X}} = 50 \pm 3\,(1.7) = 55\%$ and 45%. So if we produce a material having 50% passing the 4.75 mm sieve, we will have an allowable spread of only $\pm$ 5% instead of the special provision's permitted $\pm$ 10%. If we want to control this material then the upper and lower control limits will not be equal to the limits of the specification but the limits calculated above. U.C.L. = 55% and L.C.L. = 45%. If any of the lot means fall, say below 45%, it will be a warning that some assignable causes are acting on the process, even if the specification requirements are met.

It follows that one of the shortcomings of a fixed specification band is that the allowable tolerances for the lot means will change according to the position of the population mean.

So far we were discussing only the lot mean requirement of the special provision.

Considering the range criterion, it can be best explained by the control chart equations. Control charts are used to detect if the material is "in control" or "out of control". Variations of the material or process resulting from "chance causes" are inherent and recognized by the specification tolerances. However if an "assignable cause" acts on the process it may induce variations resulting in unacceptable material, which are detected by the charts prior to the material getting in the unacceptable zone. Production processes are usually controlled by two control charts, namely the control chart for averages and the control chart for ranges.

To construct control charts, empirical equations were formulated for two different assumptions. These assumptions are:

(1) The population parameters $\bar{X}'$ and σ' are known before the process begins.

(2) The population parameters $\bar{X}'$ and σ' are unknown before the process begins.

As explained earlier our specification indirectly specifies these parameters, thus we may use the equations for the first assumption. Since the gradation band determines the maximum acceptable standard deviation, we will use this parameter to see what would be the acceptable maximum range of any lot of the same population.

The control chart equations define the target value, the upper control limit (U.C.L.) and the lower control limit (L.C.L.). For the $\bar{X}'$ chart of subgroups of n>1 the equations are:

Target value = $\bar{X}'$

$$U.C.L._{\bar{X}} = \bar{X}' + 3\sigma_{\bar{X}}$$

$$L.C.L._{\bar{X}} = \bar{X}' - 3\sigma_{\bar{X}}$$

Since $\bar{X}$ and σ' were defined earlier, the control limits for the lot means (n=4) are:

$$U.C.L._{\bar{X}} = 45 + 3\left(\frac{\sigma'}{\sqrt{4}}\right) = 45 + 3\left(\frac{6.6}{2}\right) = 55\%$$

and

$$L.C.L._{\bar{X}} = 45 - 3\left(\frac{\sigma'}{\sqrt{4}}\right) = 45 - 3\left(\frac{6.6}{2}\right) = 35\%$$

Equations for the range chart are:

Target value = $\bar{R}$ (the mean of the lot ranges)

$$U.C.L._R = \bar{R} + 3\sigma_R \qquad (2)$$

$$L.C.L._R = \bar{R} - 3\sigma_R \qquad (3)$$

where σ_R is the standard deviation of the individual lot ranges.

There is a statistical relationship between the population standard deviation (σ') and the average subgroup range ($\bar{R}$).

$$\sigma' = \frac{\bar{R}}{d_2} \quad \text{therefore target value } \bar{R} = d_2\sigma'$$

The value of d_2 depends on the subgroup size "n" as given in statistical tables.

For $n = 4, d_2 = 2.059$ and hence

target value $\bar{R} = 2.059 \times 6.6 = 13.59 \simeq 13.6\%$

By substituting the average range $\bar{R}$ in the equations (2) and (3) and rewriting in a simplified form we have:

$$U.C.L._R = \bar{R} + 3\sigma_R = d_2\sigma' + 3\sigma_R = D_2\sigma'$$

$$L.C.L._R = \bar{R} - 3\sigma_R = d_2\sigma' - 3\sigma_R = D_1\sigma'$$

The value of D_2 and D_1 again depends on the subgroup size "n" and for n=4 $D_2 = 4.70$ and $D_1 = 0$.

Using now actual values for σ' and $\bar{R}$ we obtain:

Target value $\bar{R} = 13.6\%$

$U.C.L._R = D_2\sigma' = 4.70 \times 6.6 \simeq 31\%$

$L.C.L._R = D_1\sigma' = 0 \times 6.6 = 0$

Simplified control charts for plotting lot values (four sublots) for lot means ($\bar{X}$) and lot ranges (R) are shown in Figure 3.

By examining the R chart we will notice that the control limits are so wide that they will result in accepting totally unacceptable material. This chart allows a maximum range within one lot of 31%. We can have, for example, a lot with tests of 65%, 60%, 35% and 34% passing, where $\bar{X} = 48.5$ and $R = 31\%$. Three of the sublots are outside the specification band but the mean is within. The range chart indeed demonstrates that there is some

probability that lots having acceptable mean may have ranges as large as 31%. (The probability is rather small, less than 2.5%).

To remedy this undesirable situation we had two options to choose from. The first of these was a specification of the job mix type such as the ASTM D 2940 spec. on graded aggregates. These specifications stipulate a job mix band, within which the population mean ($\bar{X}'$) of the produced material, called the job mix formula, must lie. After the job mix formula is determined (preferably by the producer or contractor) the job mix band no longer applies, but acceptable tolerances are given by the specification about the job mix formula. The advantage of these specifications is that the same tolerances apply to any material, provided that the target value ($\bar{X}'$) lies within the job mix band. Since the distribution of the sample means (lot $\bar{X}$) is equal to the given tolerances the material is acceptable with full price even when lot means fall outside the job mix band.

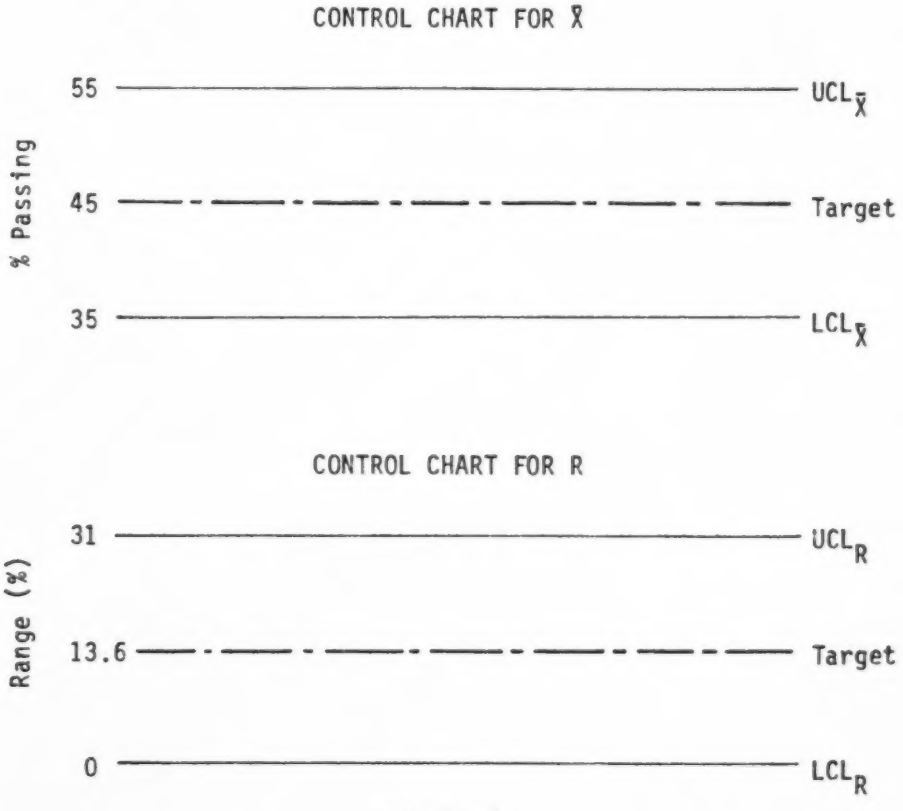


Figure 3

Control Charts for plotting Lot Means ($\bar{X}$) and Lot Ranges (R)

Job mix type specifications are truly statistical, but we must have a fairly good idea of the average gradation of the material, prior to the commencement of the work, in order to control the given tolerances. It was felt that this requirement could not be fulfilled in all of our contracts, at this time, hence this option was abandoned.

The second option available was to control the range within the lots. This solution was simpler to apply, easier to understand and also caused no drastic change in our specification. Therefore the maximum allowable ranges of measurements within the lots were established for each designated sieve. The maximum allowable ranges were calculated on each sieve by the assumption that at least 93% of all the lot means be within the specification band, even if the range of the lot is as big as the maximum allowable.

On the 4.75 mm sieve this maximum allowable range is 16%. As explained earlier there is a relationship between the average range of all the lots and the population standard deviation. Thus the dispersion is now governed by the maximum range (R_{max}).

Since it is the range that is given, we have to use control chart equations assuming that the population parameters $\bar{X}'$ and σ' are unknown.

Simplified control chart equations for this assumption are as follows:

$\bar{X}$ chart

Target value = $\bar{\bar{X}}$

$$U.C.L._{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}$$

$$L.C.L._{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$$

where $\bar{\bar{X}}$ is the mean of all the lot averages or, if not available, we may assume various $\bar{\bar{X}}$ values to determine control limits.

$\bar{R}$ chart

Target value = $\bar{R}$

$$U.C.L._R = D_4 \bar{R}$$

$$L.C.L._R = D_3 \bar{R}$$

in the above equations the factors A_2, D_4 and D_3 can again be found in statistical tables. For subgroups, n = 4 these factors are: $A_2 = 0.73$, $D_4 = 2.28$, $D_3 = 0$

Since we specified the maximum allowable range of 16%, this is equal to the upper control limit. To obtain the average range we may use the R chart equation.

$$U.C.L._R = D_4 \bar{R} = 16; \qquad \bar{R} = \frac{U.C.L.}{D_4} = \frac{16}{2.28} = 7.0\% \tag{4}$$

Knowing $\bar{R}$ and assuming various $\bar{\bar{X}}$ - s we can solve the upper and lower control limit equation.

$\bar{X}$ chart

Target value assumed $\bar{\bar{X}} = 45\%$

$$U.C.L._{\bar{X}} = \bar{\bar{X}} + A_2\bar{R} = 45 + (0.73 \times 7) = 50.1 \simeq 50\%$$

$$L.C.L._{\bar{X}} = \bar{\bar{X}} - A_2\bar{R} = 45 - (0.73 \times 7) = 39.9 \simeq 40\%$$

assumed $\bar{\bar{X}} = 50\%$

$$U.C.L._{\bar{X}} = 50 + (0.73 \times 7) = 55.1 \simeq 55\%$$

$$L.C.L._{\bar{X}} = 50 - (0.73 \times 7) = 44.9 \simeq 45\%$$

assumed $\bar{\bar{X}} = 40\%$

$$U.C.L._{\bar{X}} = 40 + (0.73 \times 7) = 45.1 \simeq 45\%$$

$$L.C.L._{\bar{X}} = 40 - (0.73 \times 7) = 34.9 \simeq 35\%$$

From the preceeding computations we can state, that by specifying the maximum allowable range of 16%, the overall average gradation (the grand mean of the population, $\bar{\bar{X}}$) may vary between 40% and 50% only, in order that all the lot averages be within the specification band of 35% and 55%. If the overall average

gradation falls in the range 35 - 40% or 50 - 55%, then some of the lot averages will not meet the specification requirement, unless the average range of the lots is less than 7%. Solving the control chart equations for ranges we have:

R chart

Target value (from equation (4) $\bar{R} = 7\%$

$U.C.L._R = D_4 \bar{R} = 2.28 \times 7 = 15.9 \simeq 16\%$

$L.C.L._R = D_3 \bar{R} = 0 \times 7 = 0\%$

A typical control chart for $R_{max} = 16\%$ is shown in Figure 4.

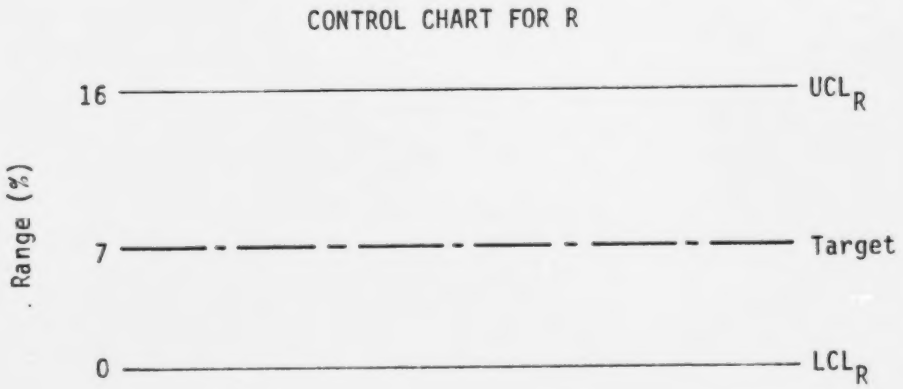


Figure 4

Having gone through the above exercise we can visualize that we have indeed a modified job mix type of specification. If we want to reformulate the requirements with statistical terms we can specify (concerning the 4.75 mm sieve requirements) that:

"The job mix band on the 4.75 mm sieve shall be 40 to 50% passing. The job mix formula shall be established at the beginning of the work and it shall specify a single percentage of aggregate passing each designated sieve. The job mix formula shall be within the limits (40-50) of the job mix band.

The lot shall be acceptable (with bid price) for gradation if the mean of the four test results is within ±5% deviation allowed from the job mix formula.

If the job mix formula falls between the limits 35-40% or 50-55% passing then the allowable deviation shall be reduced by ±1% for each percent that the formula falls outside the job mix band (See Table 1).

If the job mix formula lies within the job mix band then the maximum allowable range of the test results of a lot shall be 16%. If the job mix formula falls between the limits 35-40% or 50-55% passing then the allowable maximum ranges shall be reduced according to Table 1".

Average gradation of the source (job mix formula) % passing	Allowable maximum %		Average Lot Range $\bar{R}$ (%)
	Range within the lot	Deviations from the average:	
between 40-50	16	±5	7
39 or 51	12.3	±4	5.4
38 or 52	9.8	±3	4.3
37 or 53	6.2	±2	2.7
36 or 54	3.2	±1	1.4
35 or 55	0	0	

TABLE 1

NOTATIONS

Average gradation of the source (job mix formula) means the overall gradation (% passing) on the particular sieve. Most large commercial plants have a good idea of this. If not available we must use the best estimate available. (Precontract tests, first lot value, etc).

Deviation from the average: This is the band where all the lot means should fall. If a lot mean is outside, we should investigate the cause.

Max. range within lot: This range must not be exceeded and must not occur frequently. (Definitely not more than once in every 20 lots).

Average range: The average range of all the lots should not exceed these values.

In summary it should be now obvious that in order that all the lots of a granular A to be acceptable with full price, the average gradation of the produced material should be between the limits of 40% and 50% passing in order to utilize the maximum allowable range of 16%. (Average range of the lots should not exceed 7%).

Random checks of actual 1981 contracts on granular A acceptance confirmed that the above statistical interpretation is valid. Statistical actual parameters of some of these jobs are as follows:

Job No. 1 (4.75 mm sieve)

Population mean $\bar{X}'$ = 40.8%

Average range $\bar{R}$ = 7.1%

Maximum range R_{max} = 15.2%

Number of lots n = 31

Since the $\bar{X}'$ was between the 40-50% limits and the R_{max} < 16% all the 31 lots were acceptable.

Job No. 2

$\bar{X}'$ = 47.0%

$\bar{R}$ = 10.6%

R_{max} = 19%

n = 27%

The $\bar{X}'$ was within the limits of 40-50% but the $\bar{R}$ and R_{max} were larger than the allowable. One lot among the 27 was outside specification. This represents 3.7%.

Job No. 3

$\bar{X}'$ = 38.2%

$\bar{R}$ = 6.2%

R_{max} = 14%

n = 21

The $\bar{X}'$ was outside the 40-50% limits but within specification. R_{max} was smaller than the allowable 16% but larger than the applicable maximum for a mean at 38% (see table). Among the 21 lots, 1 lot was outside specification and four were very near (0.2 - 0.5%) the 35% limit, but within spec. This represents 4.7% non-conformance.

(3) Control limits for the rest of the designated sieves.

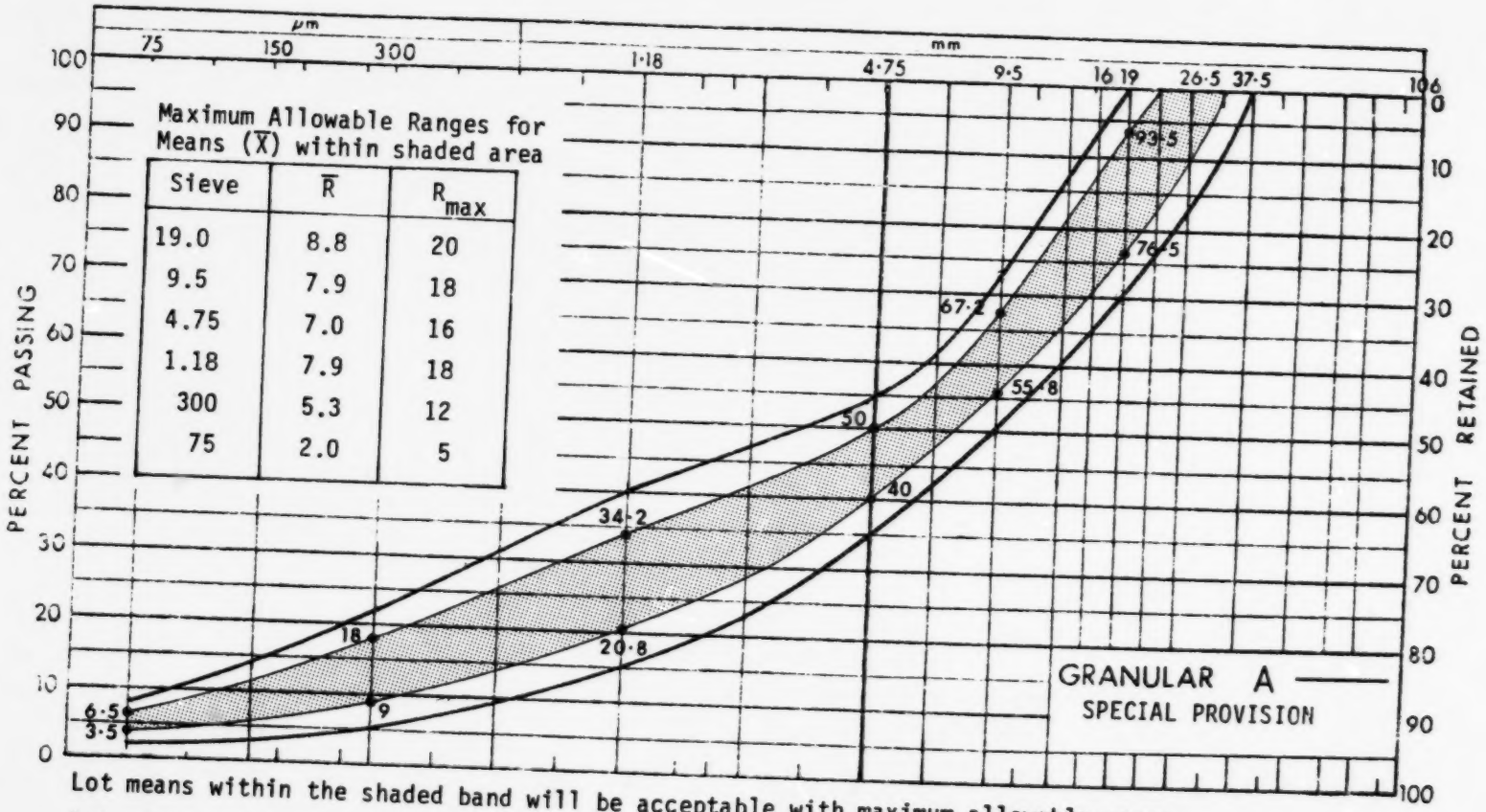
By carrying out the computations discussed under section (2) the allowable maximum deviations (tolerances) and the maximum and average ranges may be developed for all designated sieves.

The engineer (project supervisor) of course is mainly interested in the acceptance procedure. If the lot mean and range are within specification the lot is accepted with bid price. If outside, the lot is either rejected or accepted with adjusted payment. To keep the aggregate production under meaningful control (quality or process control) knowledge of the grand mean (the source average), the variations of the acceptable deviations from the grand mean, and the variations of the average and maximum ranges are essential. The control of gradation during the various phases of production (extraction, crushing, stockpiling, hauling, etc.) is the responsibility of the contractor.

It is now the policy of the Ministry that acceptance samples be taken when feasible, at the point of use. At this time however, especially at wayside pits and quarries, the MTC may still carry out some sampling at testing during production.

It is then equally important for both parties, contractors (aggregate producers) and MTC construction personnel to understand the control limits and requirements of the aggregates. In Figure 5 the necessary statistical control limits are marked on a gradation chart. This will assist the engineer and contractor (producer) to control production and detect any assignable cause acting on the process.

In the gradation chart the shaded area within the specification band shows the zone within which the grand average gradation of a particular source ($\bar{\bar{X}}$) must fall in order to utilize the allowable maximum range criterion for acceptance with full price. It is to be realized that the average range of the lots ($\bar{R}$) must be much smaller than the maximum allowed by the special provision even if the $\bar{\bar{X}}$ is within the shaded zone.



Lot means within the shaded band will be acceptable with maximum allowable range.

Lot means outside the shaded band but inside the specification band should have reduced range.

Figure 5

Values of $\bar{R}$ of the lots are marked in the gradation chart. If the actual $\bar{R}$ of the lots is larger than the ones marked, then some of the lots will be outside the range criterion for full price acceptance.

If the average gradation of a source falls outside the shaded area but inside the upper and lower limits of the specification band, then the maximum allowable ranges must be reduced in order that all the lots be accepted with full price.

In Table 2 the reduced range requirements are given for the source grand means that fall within the specification band but outside the shaded area.

Average Gradation (% from the specification limits but inside the band) **	Allowable maximum range (R_{max}%)	Average lot range ($\bar{R}$%)
1	3.1	1.4
2	6.2	2.7
3	9.4	4.1
4	12.5	5.5
5	15.6	6.8
6 *	18.6	8.2

TABLE 2

* 19.0 mm sieve only.

** e.g.: If the spec. limits is say 5-22%, 1% means that the average gradation $\bar{\bar{X}}$ is 5+1 = 6% or 22-1 = 21%. If this is the case then the average range of the lots should not exceed 1.4%.

If a source produces granular A with an average of 53% passing the 4.75 mm sieve the lot means may vary $\pm$2%, the average lot range must not be greater than 2.7% and the maximum range must not exceed 6.2%. If the above conditions are not met, then some of the lots will be outside the specification requirements.

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Willenbrock, J.H., (1976); *A Manual for Statistical Quality Control of Highway Construction;* Volume 1 and 2, FHWA, Washington, D.C.

APPENDIX I

GRANULAR 'A' - Item No

Acceptance of Granular 'A'

Description

The Ministry will determine the acceptability of Granular 'A' gradation and percent crushed particles on a lot-by-lot basis as outlined in this special provision. When either the Granular 'A' gradation or the percent crushed particles is not within the Ministry's specification limits, an adjusted payment will be allowed at the request of the Contractor in lieu of rejection, except where the non-conformance is so great that the material is totally unacceptable.

References

MTC Laboratory Testing Manual.

Definitions

LOT	- a specific quantity of material normally from a single source, or a specific amount of construction assumed to be produced by the same process.
SUBLOT	- approximately equal divisions or portions of a lot.
MEAN	- the arithmetic average of a set of data.
RANGE	- the difference between the largest and smallest measurement in a set of data.
RANDOM SAMPLE	- a sample chosen in such a manner that each portion in the sublot has an equal probability of being selected.
RANDOM NUMBER TABLE	- a set of numbers generated by chance.

Lot Size

The rate of sampling shall be based on lots. The maximum lot size shall be 4000 tonnes of material; however, the Engineer may increase the lot size, if the previous four lots have met the specification requirements, to 10,000 tonnes.

The lot shall be divided into four approximately equal sublots and one sample shall be selected on a random basis from each sublot.

In the event that operational conditions cause work to be interrupted before the lot has been completed, the Engineer may determine the acceptability of the incomplete lot on the basis of the test results of two or three sublots.

Where it is necessary to designate the quantity of material in a lot or part of a lot, for the purpose of payment adjustment, the Engineer's estimate of this quantity shall be used.

Sampling procedure

The Ministry will perform all necessary sampling and testing for acceptance purposes.

Sampling may be carried out at the source during production, from existing stockpiles, from the material being loaded on delivery trucks, or from the roadway.

Samples taken during production will be taken as the material is crushed and stockpiled.

Samples may be taken from existing stockpiles containing 4000 tonnes of material or less. The Engineer may permit larger stockpiles to be sampled after sub-dividing or otherwise reworking by and at the expense of the Contractor.

Delivery or road samples will be used when the Engineer decides that sampling at the source or from existing stockpiles will not ensure that the material sampled is representative of that to be delivered to the job site.

Where sampling at the source during production or at the time of delivery is used, samples may be taken at random times related to the amount of material being produced. Where existing stockpile or road sampling is used, samples will be taken from tonnages selected at random or from areas selected at random. Regardless of the method of sampling followed, tables of random numbers will be used.

The Contractor will be advised in writing, prior to the use of delivery or road sampling. Samples taken from material already placed in the work, other than retest samples, shall be taken prior to compaction, for which the Contractor shall provide reasonable opportunity.

Testing

Gradation testing of the samples will be performed by the Ministry as described in MTC Laboratory Testing Manual, Test LS-602 "Method of Test for Sieve Analysis of Fine and Coarse Aggregates".

Gradation test results will be computed for each sieve designated in Table 1.

Testing for percent crushed particles will be performed by the Ministry as described in MTC Laboratory Testing Manual, Test LS-607, "Method of Test for Determination of Percent Crushed Particles in Processed Coarse Aggregate".

Tests from production, stockpile and road samples will not be mixed in considering the acceptance of any particular lot, except where necessary when retesting.

Specification Requirements

(i) Gradation

(a) Completed Lots

A lot of Granular 'A' will be deemed to meet the specification requirements for gradation if the mean of the four test results of the lot is within the gradation limits specified in Table 1 and the range of the test results of the lot is equal to or less than the maximum acceptable range in Table 1.

(b) Incomplete Lots

In the event that the Engineer decides that acceptance will be determined on the basis of an incomplete lot, the incomplete lot shall be deemed to meet the specification requirements for gradation if the mean of the results is within the gradation limits in Table II and if the range of the tests is within the maximum range given in Table 1.

TABLE 1

GRADATION REQUIREMENTS - FOUR SUBLOTS

Ministry Sieve Designation	% Passing by Mass Four Sublots	Maximum Acceptable Range
37.5* mm	100	0
19.0 mm	70 - 100	20
9.5 mm	50 - 73	18
4.75 mm	35 - 55	16
1.18 mm	15 - 40	18
300 um	5 - 22	12
75 um	2 - 8	5
	2 - 10**	5**

* Where the contract requires the maximum size to be 22.4 mm, the percentage passing that sieve shall be 97-100 and the maximum acceptable range shall be 3%.

** Where the Granular 'A' is obtained from a quarry source.

TABLE II

GRADATION REQUIREMENTS - LESS THAN FOUR SUBLOTS

Ministry Sieve Designation	% Passing by Mass Two Sublots	Three Sublots
37.5* mm	100	100
19.0 mm	63 - 100	67 - 100
9.0 mm	44 - 79	47 - 76
4.75 mm	30 - 60	33 - 57
1.18 mm	9 - 46	12 - 43
300 um	1 - 26	3 - 24
75 um	1 - 9	1.5 - 8.5
	1 - 11**	1.5 -10.5**

* Where the contract requires the maximum size to be 22.4 mm, the limits for that sieve for two and three sublots shall be 96-100 and 96.5-100 respectively.

** Where the Granular 'A' is obtained from a quarry source.

(ii Percent Crushed Particles

A complete or incomplete lot of Granular 'A' shall be deemed to meet the specification requirements for percent crushed particles if the mean of the test results is equal to or greater than the applicable value shown in Table III, and no individual sublot has a test result less than 45%.

TABLE III

Number of Sublots	Percent Crushed Particles
Two	47
Three	49
Completed Lot (Four)	50
Retested Lot	52

(iii) Visual Examination

Notwithstanding the above testing of the Granular 'A', any load which is observed visually by the Inspector to be outside the specification requirements shall be rejected. Should the Contractor require, rejection shall be confirmed by taking two random samples from the load and applying the requirements for an incomplete lot with two sublots. Where this testing confirms that the load qualifies for rejection under Section (iii) of the Basis of Payment, the Contractor shall be charged the cost of retesting at half the rate shown in Section (iv), Retesting. In all other cases the cost of sampling and testing shall be borne by the Ministry and the load shall be decisioned as part of the lot.

Basis of Payment

(i) Payment at the Contract Price

Payment for complete and incomplete lots which meet the specification requirements will be made at the contract price for Granular 'A'.

(ii) Adjusted Payment

Payment for complete or incomplete lots which do not meet the specification requirements but which are not subject to rejection under (iii) following, will be adjusted as follows:

(a) Gradation

If the Contractor chooses to use a lot of Granular 'A' which does not meet the specification requirements for gradation, or where such a lot cannot be totally excluded from the work, payment for that lot shall be made at a price equal to the contract price for Granular 'A' less a percentage equal to the sum of the adjustment points determined on the following basis: For each 0.1% that the mean gradation falls outside the specified limits for each sieve, adjustment points shall be applied according to Table IV.

TABLE IV

Ministry Sieve Designation	Adjustment Points per 0.1% Deviation from Specification Limits
37.5* mm	0.1
19.0 mm	0.1
9.5 mm	0.1
4.75 mm	Excess Passing 0.5 Insufficient Passing 0.2
1.18 mm	0.1
300 um	0.1
75 um	1.0

* Where the contract requires the maximum nominal size to be 22.4 mm, the adjustment for the 37.5 mm sieve shall apply to the 22.4 mm sieve.

For each 0.1% that the range exceeds the maximum acceptable range for each sieve, shown in Table I, 0.1 adjustment points shall be applied.

(b) Percent Crushed Particles

If the Contractor chooses to use a lot of Granular 'A' which does not meet the specification requirements for percent crushed particles, or where such a lot cannot be totally excluded from the work, payment for that lot shall be made at a price equal to the lesser of:

- the contract price for Granular 'A' less 5% for each percent that the lot average falls below the applicable limits shown in Table III, or

- the contract price for Granular 'A' less 5% for each percent that any one individual sublot falls below 45% (47% in the case of a retest).

(c) Gradation and Percent Crushed Particles

If the Contractor chooses to use a lot of Granular 'A' which does not meet the specification requirements for gradation, and which also does not meet the specification requirements for percent crushed particles, or where such a lot cannot be totally excluded from the work, payment for that lot shall be made at a price equal to the contract price for Granular 'A' less the sum of the percentages prescribed in (a) and (b) above.

(iii) Rejection

Complete or incomplete lots which are either

- subject to a price adjustment of more than 25% of the contract price in respect of gradation only, or

- subject to a price adjustment of more than 25% of the contract unit price in respect of percent crushed particles only,

shall be rejected and shall not be incorporated in the work, or, where already incorporated shall be removed from the work.

(iv) Retesting

Upon receipt of the test results on any lot the Contractor may require in writing that the lot be retested, either for gradation or for percent crushed particles, or for both. It shall then be the responsibility of the Contractor to identify and make available the original sublots for further sampling.

Where this notice is given and these conditions are fulfilled the Ministry will carry out retesting by taking an additional random sample from each sublot and testing it for gradation, percent crushed particles, or both, as designated by the Contractor.

The lot will then be re-decisioned for payment or rejection as in (i), (ii) and (iii) above, except that the following values will be used:

For gradation limits - the average of all the test results (original and retest)

For gradation range - the average of the original and the retest ranges

For percent crushed, lot average - the average of all the test results (original and retest)

For percent crushed, individual sublots - the average of the original and the retest results for each sublot

and except that the gradation limits shown in Table V shall be used in place of those shown in Table I.

No lot shall be retested more than once.

TABLE V

GRADATION REQUIREMENTS - FOUR SUBLOTS

TWO SAMPLES PER SUBLOT

Ministry Sieve Designation	% Passing by Mass
37.5* mm	100
19.0 mm	75 - 95
9.5 mm	52 - 71
4.75 mm	37 - 53
1.18 mm	17 - 38
300 um	7 - 20
75 um	3 - 7
	3 - 9**

* Where the contract requires the maximum size to be 22.4 mm, the percentage passing that sieve shall be 97.5-100.

** Where the Granular A is obtained from a quarry source.

Where the retesting causes a lot which had been subject to rejection or to acceptance at an adjusted price, to be accepted at the contract price, the cost of the additional sampling and testing shall be borne by the Ministry.

In all other cases the Contractor shall be charged the cost of retesting as follows:

Gradation only	$200 per lot
Percent Crushed Particles only	80 per lot
Gradation and Percent Crushed Particles	280 per lot

(v) Delay to Work

All costs arising from any delay whatsoever to the work which may be occasioned by the procedures required herein shall be deemed to be included in the contract price for this item.